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# ParaDiS on BlueGene/L: scalable line dynamics

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#### Abstract

We describe an innovative highly parallel application program, ParaDiS, which computes the plastic strength of materials by tracing the evolution of dislocation lines over time. We discuss the issues of scaling the code to tens of thousands of processors, and present early scaling results of the code run on a prototype of the BlueGene/L supercomputer being developed by IBM in partnership with the US DOE's ASC program.

# Introduction to dislocation dynamics

The theory of crystal plasticity came to existence in the late nineteenth century [1]. With time it has become a proving ground for many novel methods of applied and computational mathematics. While very useful in practical applications in structural mechanics, geophysics, and other areas, mathematical crystal plasticity developed into a vestige of phenomenology, having little or, sometimes, no connection to the underlying microscopic physics of material behavior. Yet at the same time, metallurgists and physicists continued to ask hard questions: *why* do crystals deform in the ways they do and *what are the mechanisms* by which plasticity comes about?

Realization that the plastic strength of crystalline materials is controlled by the motion of its line defects, dislocations, has firmed up in stages. In 1934, the concept of dislocations as physical agents of crystal plasticity was proposed, simultaneously and independently, by three eminent scientists – Taylor, Polanyi and Orowan. While essentially explaining most of the puzzling phenomenology of crystal plasticity, dislocations still remained only a beautiful hypothesis until the late 1950's when first sightings of them were reported in electron microscopy experiments. Since then, the ubiquity and importance of dislocations for crystal plasticity and numerous other aspects of material behavior has been regarded as firmly established as, say, the role of DNA in promulgating life.

Having largely answered why crystals deform as they do, dislocations provide a natural basis for bringing physical content into the mathematical theory of crystal plasticity. However, blocking the road to a grand unification of dislocation physics and continuum crystal plasticity is a very serious hurdle: in order to be representative of macroscopic plasticity behavior, the motion and interaction of dislocations *en masse* must be considered. Given that the physics of dislocation motion and interactions are very well understood, the nature of this obstacle is purely computational: one needs to be able to

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trace the simultaneous evolution of millions of dislocation lines over extended time intervals, in order to directly *compute*, as opposed to *fit*, the plastic strength of crystalline materials. This is one exemplary case where *the size does matter*. A mesoscopic approach of Dislocation Dynamics attempts to address this challenge.

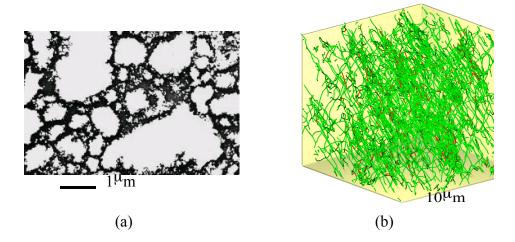


Figure 1. (a) Dislocation microstructure developed in copper during plastic deformation, as observed in an electron microscope [2]. The dark regions contain large numbers of dislocation lines whereas the light regions contain no dislocations. (b) Dislocation microstructure developed during plastic deformation in a dislocation dynamics simulation by ParaDiS.

### Dislocation Dynamics: unifying dislocation physics and crystal plasticity

The idea behind the computational approach of Dislocation Dynamics (DD) is simple. One introduces dislocation lines in the computational volume, letting them interact and move in response to forces inflicted by external loads and inter-dislocation interactions. The accuracy of DD simulations is assured by dislocation physics and mechanics that supply methods for computing forces acting on dislocations and dislocation velocities under these forces. Although the DD approach was introduced relatively recently, in the early 1990s, its potential impact on the science of material strength is well recognized [3-5]. Yet, despite multiple pronouncements of inevitable success, the DD approach has not come close to the levels of computational performance required to demonstrate that it can, indeed, predict macroscopic strength from the collective behavior of dislocations. Given the computational complexity of the problem at hand, the limited success DD can claim so far is not surprising. At the same time, the performance levels required for "break-through" DD simulations can be very well quantified.

Among various challenges worthy of large-scale computational attack, understanding the nature of strain hardening and dislocation patterning in metals is one of the most famous and holds a special value among researchers in the area of material strength. If direct DD simulations can be shown to accurately reproduce dynamic hardening transitions that

occur naturally during crystal deformation<sup>2</sup>, even skeptics will be convinced that the microscopic physical theory of crystal strength has arrived in the form of DD. With this in mind, we decided to gear our new DD code towards a single large-scale "hero" simulation that will cover the length and time scales sufficient to observe the hardening transitions that occur naturally, as a result of collective motion and rearrangement of dislocations. Careful estimates show that to be able to naturally account for hardening and dislocation patterning and avoid "small volume" artifacts, the model should include from 1M to 100M dislocation segments. Furthermore, the evolution of such large dislocation groups will have to be traced over millions of time steps, to reach the strain levels at which the hardening transitions are observed. DD capabilities available up to now at LLNL and elsewhere stop short of these target performance figures by some 2-3 orders of magnitude. Massively parallel computing, such as will be available with BlueGene/L in 2005, is the only viable pathway to closing this performance gap.

There are several challenges to this program that can be formulated at the outset. The first one is a programming challenge: DD models deal with the ever-changing topology of dislocation lines and line networks requiring rather complex data structures. On parallel machines, this challenge is dramatically amplified due to the need to handle the topology of these line networks across the boundaries of computational domains. The second challenge is a natural tendency of dislocation lines to cluster in space (owing to the long-range interactions among the lines) and develop highly heterogeneous distributions of degrees of freedom making it difficult to achieve a good load balance. The third challenge is to handle multiple time scales observed in the evolution of large dislocation ensembles, where periods of relative calm (slow motion of lines) are interspersed with bursts of very high activity over which small groups of lines moves very fast and experience multiple collisions. The code ParaDiS recently written at LLNL has shown much promise in addressing these challenges.

In our recent simulations we identified an interesting strategy for planning and executing the simulations. In the course of deformation, dislocations multiply, increasing their numbers by 2-3 orders of magnitude. For this reason, it is possible and sufficient to start with a relatively small model and let it grow on a small machine. Then, following a steadily growing number of dislocations, the job should be moved to progressively larger machines. This sort of progression worked very well when we scaled our simulation up from 12 to 100, then to 200 and, eventually, to 1,500 CPUs on MCR. The same strategy is now applied on the growing Blue Gene /L machine, which is planned to have 131,072 processors when delivered to LLNL in early 2005.

## The ParaDiS project

*ParaDiS* stands for *Parallel Dislocation Simulator*. It is a massively parallel dislocation dynamics simulation code that is being developed at the Lawrence Livermore National Laboratory since 2001. ParaDiS is specifically designed for investigating the collective behavior of large numbers of dislocation lines as required for understanding and accurate

<sup>&</sup>lt;sup>2</sup> Strain hardening is responsible for some well-known facts of everyday life, such as why an aluminum paper clip eventually breaks after bending it back and forth several times.

prediction of plasticity and strength in crystals. The code is mainly written in C and uses the MPI library for communication among the processors.

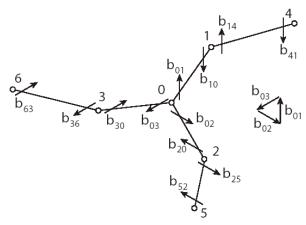


Figure 2. A fragment of the dislocation lines network: each line segment xy carries a unit of "vector current" quantified by Burgers vector  $b_{xy}$ .

The object of a ParaDiS simulation is a network of nodes connected by line segments. A node can have two or more segments or arms connecting it to neighboring nodes. Each line segment carries a unit of "vector current" or Burgers vector. Similar to the scalar currents in electric circuits, Burgers vectors are conserved along the lines and satisfy the Kirchoff rule: the sum of Burgers vectors of all arms emanating from a given node is zero, e.g.  $\mathbf{b}_{01} + \mathbf{b}_{02} + \mathbf{b}_{03} = 0$  (see Fig. 2). Also similar to electric current, the network can never terminate on a node with a single arm that carries a non-zero Burgers vector. During the simulation, the network topology evolves as new nodes are added and some old nodes get deleted. The rest of the nodes move in space in response to the forces they see.

#### Nodal forces and motion

The nodes move in response to nodal forces according the first order equation of (over-damped) motion:

$$\frac{d\vec{r}_i}{dt} = M[\vec{f}_i]$$

$$\vec{f}_i = -\frac{\partial E[\{\vec{r}_i\}]}{\partial \vec{r}_i}$$

where  $f_i$  is the force on node i, E is the energy of the dislocation network [1], and  $M[f_i]$  is a mobility function giving the velocity of node i as a function of nodal force  $f_i$ . The network energy E includes the interaction between all network segments and between the segments and applied stress. The segment-to-segment interaction forces are long range, taking the lion's share, typically more than 80%, of the total CPU time to compute.

#### **Topological rearrangements**

In addition to moving the nodes, ParaDiS evolves the network topology, to reflect the physics of dislocation motion and collisions in real crystals. Handling the evolving topology of moving and intersecting lines is a major bookkeeping challenge, especially in a parallel implementation of dislocation line dynamics. It is therefore highly desirable to reduce logical complexity of the topological switches. Presently, ParaDiS relies on two basic operations: 1) insert a new node and 2) merge two nodes in one. Whenever two nodes become doubly connected (Fig. 3(c)), the double-arm is replaced by a single arm with the Burgers vector equal to the sum of the Burgers vectors of its two parents. Yet even this simple algorithm requires considerable care since the nodes participating in a single topological rearrangement may well belong to different processors. Even though topological changes consume only a small fraction of the computing time, the associated logic and bookkeeping constitute about 50% of the ParaDiS source code.

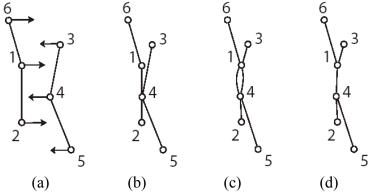


Figure 3. Handling topological changes caused by dislocation collisions: (a) just before the collision; (b) at the collision time a new node is inserted in segment 1-2 and merged with node 4; (c) a similar insert-and-merge sequence involving node 1 and segment 3-4 (d) the double arm connecting nodes 1 and 4 is reduced to a single arm.

## Spatial domain decomposition and dynamic load balancing

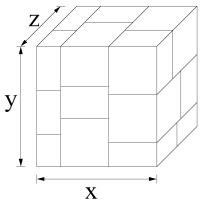


Figure 4. Example decomposition of 3-D simulation space into 3×3×2 domains.

ParaDiS relies on spatial decomposition to partition and distribute the nodal data and computational load over the processors. This is essential for good scalability, since most communications take place between neighboring processors. A common approach to data decomposition is to partition the entire simulation space into equally sized

rectangular domains and to assign each domain to a processor. Unfortunately, this approach leads to poor scalability due to a tendency of dislocation line dynamics simulations to accumulate load imbalance. This is because in the course of simulations, the lines tend to organize themselves spontaneously into spatially heterogeneous (lumpy) arrangements. Associated large variations of the local line density cause computational loads to vary significantly from one processor to another. To maintain scalability ParaDiS recursively partitions the problem domain first in the X, then in the Y, and finally in the Z dimension. The X-partition assigns equal computational load (weight) to each YZ slab. Each slab is then independently partitioned in the Y direction into blocks of equal weight, and similarly for the Z direction. At regular intervals, ParaDiS reevaluates the computational load and shifts the domain boundaries to maintain a good balance.

## Algorithm cycle and communication patterns

Because dislocation interaction is long range, any two line segments interact with each other in a ParaDiS simulation. For computational efficiency, all segment-segment interactions are partitioned into *local* and *remote* contributions, based on proximity of the interacting segments. For this purpose, a static mesh of cubic *cells* is overlaid on the problem space. The interactions between two segments in the same or in the neighboring cells are considered and treated as *local*. Otherwise, they are considered *remote*. The *local* interactions are computed explicitly for each local segment pair, while the effect of all *remote* segments in a single cell are lumped together into a super-segment contribution.

During a given algorithm cycle, *local* and *remote* forces on each network node are computed, node positions are advanced, and topological rearrangements are performed, if necessary. A single ParaDiS cycle consists of the following substeps<sup>3</sup>.

- 1) Each node is assigned to a cell according to its spatial location.
- 2) Each domain sends/receives the state (including connectivity) of each node assigned to the cells neighboring the domain's own cells. The nodes received from the neighboring domains are called *ghost nodes*.
- 3) To account for the remote forces, all segments in a given cell are lumped into a single net super-segment and its net charge is communicated via a *global* reduction (communication) operation to all domains.
- 4) Total force on each node is computed as a sum of *local* and *remote* interactions.
- 5) Nodal velocity is calculated, based on the nodal force and the mobility function. The velocities are propagated to ghost nodes via *local* communication.
- 6) Nodal positions are advanced. Line collisions are detected and the topology adjusted accordingly. Any topological changes are propagated via *local* communication.

<sup>&</sup>lt;sup>3</sup> In the following, a *local* communication is between the neighboring processors (domains) while in a *global* communication, all processors talk to each other. ParaDiS tries to minimize the amount of global communication as much as possible.

- 7) The lines are remeshed: nodes added and removed as line geometry requires. Changes are propagated via *local* communication.
- 8) Occasionally, load balance is re-evaluated based on the times it takes each processor to compute the nodal forces. The results are used to adjust domain boundaries for improved load balance. A small amount of *global* communication is required.
- 9) Nodes that migrated across domain boundaries due to nodal motion and/or domain boundary adjustments are transferred to new domains, via *local* communication.
- 10) I/O for this cycle is performed, potentially requiring some *global* communication.

During a single cycle, *local* communications take place several times. At any particular time, a given domain overlaps a set of cells, its *native* cells. Since the cell mesh is regular, each cell has 26 neighboring cells. Any other domain that overlaps either a native cell or a neighbor of a native cell of the given domain, is considered to be a neighbor domain. In most of the local communications (the ghost node communication of step 2 is the exception) only a small amount of data is actually transferred. This communication pattern of ParaDiS fits well with the architecture of BG/L, with its 3-D torus network for the neighboring processors and a separate tree structure for global communications. Note, however, that due to the somewhat irregular spatial decomposition of ParaDiS domains, nearest neighbor domains needn't always correspond to nearest neighbor processors in BGL's torus network, though on average they will tend to be nearby.

# Typical output of ParaDiS

Like any other simulation program, ParaDiS has access to all of its degrees of freedom at every time step. This information is written out periodically and is used to visualize the evolution of the dislocation network, as in Fig. 1(b). In a typical ParaDiS simulation, the initial distribution of dislocations is deformed at a constant strain rate (e.g. 1 s<sup>-1</sup>). Two important outputs from this type of simulation are stress, Fig. 4(a), and total dislocation density, Fig. 4(b), as functions of strain.

During a simulation, the dislocation line density and, hence, the number of nodes increase by 1-2 orders of magnitude. This multiplication behavior is typical of crystals under stress. An important computational implication is that the size of the problem (the number of network nodes) steadily increases as the simulation proceeds. As is very well known to every dislocation simulator, even a small model of the dislocation network that accumulates strain briskly in the beginning of the simulation, quickly outgrows the computer it ran on bringing the simulation nearly to a standstill. As an example, the results shown in Fig.5 were obtained in a simulation that initially ran on a 12 CPU Linux cluster but was later moved to 200 CPU Linux cluster and, finally, to 512 CPUs of an IBM-SP3 machine. We anticipate that, by the time the BG/L machine grows to its full size of 64K nodes, the line dynamics simulation we report on here will also grow to a size that demands a machine of that size.

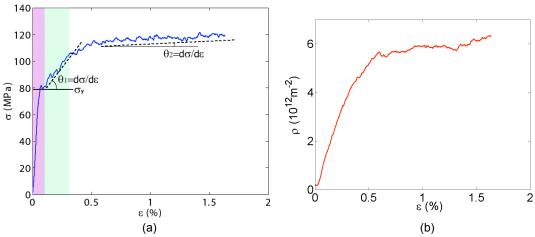


Figure 5. (a) Mechanical strength of single crystal molybdenum computed in a single ParaDiS simulation. The red region is a typical amount of strain a serial dislocation code can accumulate. The blue region shows how much strain ParaDiS was able to generate a year ago. Currently, ParaDiS can generate more than 3% of strain in a single simulation. Variations in the slope of the stress-strain curve reflect dynamic transitions in the collective behavior of dislocation lines. (b) The same transitions are also seen in the behavior of the total density of dislocation lines as a function of strain.

# The BlueGene/L ASC platform

BlueGene/L is a next-generation massively-parallel computing system designed for research and development in computational sciences. BlueGene/L is targeted for delivering hundreds of teraflops to selected applications of interest to the US Department of Energy/National Nuclear Security Agency's Advanced Simulation and Computing program (ASC). It is an extremely high compute-density system with very attractive cost-performance and relatively modest power and cooling requirements. BGL is being developed by IBM in partnership with ASC, and is planned to be delivered to Lawrence Livermore National Laboratory in early 2005. Currently there exist a 4096-node 500 MHz prototype using pass 1 chips, and a 512-node 700 MHz system with pass 2 chips.

BlueGene/L has been extensively described elsewhere [6,7]. As can be seen in Table 1, BGL is substantially different from previous ASC supercomputers. The 65,536 compute nodes of BGL are each composed of just a single moderately-clocked chip, together with their attendant main memory chips, drastically lowering power consumption and space requirements, while favoring communication and memory performance. The BGL chip contains two independent processors, each capable of two floating point operations per cycle (including fused multiply-adds, yielding a theoretical peak of 4 floating point operations per cycle), several independent network controllers, three levels of cache (including 4 MiB L3), and memory controllers. Though each floating point unit is capable of two operations per cycle, they are not independent: the second floating point pipe is usable only by 2-way SIMD instructions, or by 2-way "SIMOMD" (i.e., "single instruction, multiple operation, multiple data") instructions.

The two processors on each chip are identical, with symmetric access to resources. It is

expected that most applications will use *communication coprocessor mode*, running a single MPI task per node, with one processor running the application, but with the MPI library offloading much of the work of message passing to the second processor. Applications may also run in *virtual node mode*, running two MPI tasks on each node (one on each processor), or in *dual-core mode*, using a fork-join mechanism to perform computation on the second processor (in which case the user must deal explicitly with the lack of coherence in the L1 caches).

MPI communications are handled by three independent special-purpose networks in BGL. Point-to-point and all-to-all communications are handled by a 32x32x64-node three-dimensional torus, with each node connected to its nearest neighbors via six independent bidirectional links. In addition to the torus, BGL also has tree networks to perform global operations like broadcasts, reductions, and barriers with very low latency and high bandwidth, e.g., the entire 65,536-node machine is targeted to complete an MPI AllReduce in less than 10 microsec and an MPI Barrier in 5 microsec.

	White	Q	EarthSim	MCR	RedStorm	BGL
Peak Tflops	12.3	20	40	11.2	40	~360
Install date	2000	2000	2002	2002	2004	2005
Total memory (TiB)	8	22	10	4.5	10	32
# nodes	512	4096	640	1152	10,368	65536
Cpus/node	16	4	8	2	1	2
Proc. freq. (MHz)	375	1000	500	2,400	2,000	~700
Peak/node (Gflops)	24	7.3	64	9.7	4.0	~5.6
Mem/node (GiB)	16	8	16	4	1	0.5
Mem latency (cycls)	140	330	-		~100	75
Total mem BW (TB/s)	8	19	160	3.7	55	360
Mem Bytes/flop	0.65	0.95	4.00	.33	1.4	1.0
Total comm BW	0.5	1.7	5.2	.8	187 (?)	270.0
(TB/s)						(torus)
Comm Bytes/flop	0.04	0.09	0.13	0.07	4.7 (?)	0.75
						(torus)
Bisection BW (TB/s)	0.5	0.8	1.2	0.13	2.3	2.9
						(torus)
Footprint (1000 ft <sup>2</sup> )	10	20	34		3.0	2.5
Gflops/ft <sup>2</sup>	1.23	1.00	1.18		13	144.0
Total power (MW)	1.0	3.8	10		1.7	1.5
Tflops/MW	12.3	5.3	4.0		23.5	240.0

Table 1. A comparison of recent and near-future ASC platforms. [Note: empty boxes will be filled in the final version of the paper.]

## **Performance results**

In the performance results presented below, we have used strong scaling to compare the performance between 32 and 512 processors of BlueGene/L (using the 700 Mhz prototype) and MCR. By using strong scaling, we ensure that the problems on the 32- and

512-processor systems are directly comparable. On BGL we ran in communications coprocessor mode. The current version of the IBM xl compiler does not generate SIMD instructions for the "double hummer" floating point unit, and levels of optimization higher than –O2 were unavailable at the time of these runs. MPI tasks were not explicitly mapped to BGL's torus in these experiments, i.e., the MPI runtime implementation's default assignment of tasks to processors was used. [Note: We expect to have results from up to 2048 or 4096 BGL nodes in time for inclusion in the second version of this paper, if conditionally accepted. We also expect to have weak scaling results available for the second version.]

Figure 6 shows the scaling performance expressed as the so-called "grind time," calculated as  $(\Delta t \cdot numtasks)/(numsteps \cdot numdislocnodes)$ , where  $\Delta t$  is the elapsed time from the start of cycle 76 to the end of cycle 100 (so *numsteps* is 25). (These cycles were chosen because the initial 75 cycles are atypical.) Perfect scaling would yield a constant grind time. Given that the peak flops rate on an MCR processor is more than 1.5x that of a BGL processor, and given the pre-production state of the BGL compiler, BGL's absolute performance compares surprisingly well with MCR.

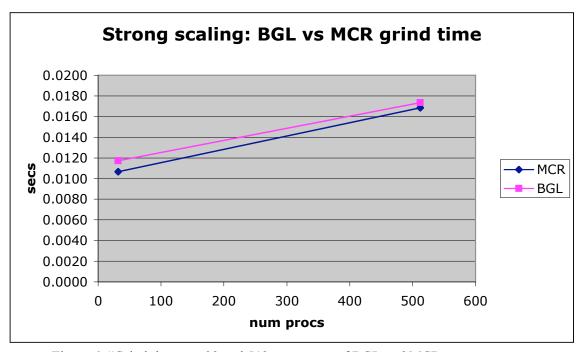


Figure 6. "Grind time" on 32 and 512 processors of BGL and MCR.

Figure 6 shows that the scaling behavior of this problem is almost identical on BGL and MCR within this range of processor counts. As suggested by the detailed substep timings shown below, we think the scaling on BGL may improve if tasks are explicitly mapped to to BGL's torus network so as to minimize the torus distance between logically nearest-neighbor domains, but this is left for future investigation, as is measurement at larger numbers of processors as they become available.

Figure 7 and Tab. 2 show times of the individual subparts of step 100 on 512 tasks. By far the greatest time is spent in the nodal force calculation, which is computationally intensive, containing no communication. Communication is faster on BGL, except for comm\_send\_ghosts, which contains the only large point-to-point message; future experiments will determine whether this can be ameliorated by a better mapping of MPI tasks to compute nodes in the BGL torus network. Cell\_charge contains significant global communication, which is expected to give BGL a significant advantage at large processor counts, but has similar time on both BGL and MCR at these processor counts here.

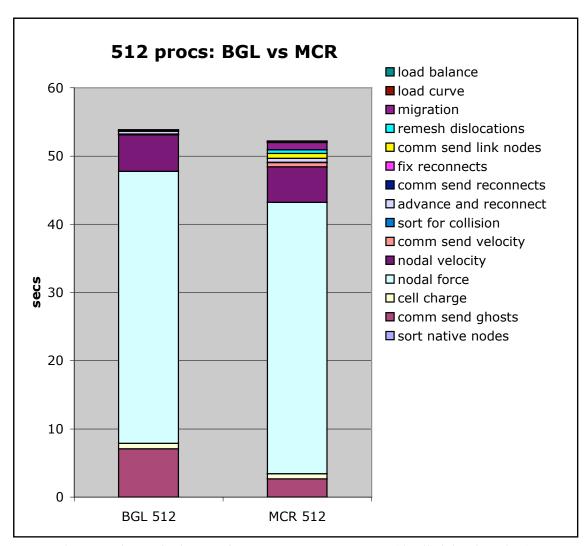


Figure 6. Times of subparts of step 100 on 512 processors (detailed data in Tab. 2).

	BGL	MCR
sort native nodes	0.001	0.001
comm send ghosts	7.098	2.674
cell charge	0.789	0.769
nodal force	39.867	39.755
nodal velocity	5.352	5.228
comm send velocity	0.046	0.619

sort for collision	0.061	0.047
advance and reconnect	0.407	0.583
comm send reconnects	0.001	0.002
fix reconnects	0.000	0.000
comm send link nodes	0.053	0.699
remesh dislocations	0.041	0.527
migration	0.098	1.062
load curve	0.024	0.087
load balance	0.026	0.146

Table 2. Times of subparts of step 100 on 512 processors (plotted in Fig. 6).

Table 3 shows the overall load imbalance (percentage of time spent waiting) achieved by ParaDiS on the studied problem over 25-step intervals. During the first 75 steps, load becomes increasingly balanced, then stays fairly balanced.

	1-25	26-50	51-75	76-100	101-125	126-150
BGL	40%	25%	20%	6%	9%	6%
MCR	39%	25%	19%	6%	10%	9%

Table 3. Load imbalance over 25-step intervals on 512 processors.

[Note: future investigations which we believe can be included before the final abstract deadline and final paper deadline include: more processors, explicit mapping of tasks to BGL torus, weak scaling, virtual node mode and/or dual-core mode, flops counts.]

#### **Summary**

A new code for line dynamics simulations, ParaDiS, is being developed at LLNL. The code shows considerable promise to deliver the long-awaited breakthrough in the computational materials sciences: a first direct calculation of material strength based on the underlying behavior of dislocation lines. The algorithm and the communication pattern of ParaDiS match well the architecture of BGL. As more capable machines become available, so grows the modeled dislocation network, promising to reach the target level of complexity through efficient utilization of the new computational resources.

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